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Inverse Optimization: A Case Study in Supply Chain Cost Estimation

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ABSTRACT

Determining unknown parameters of an optimization model such that a given feasible solution becomes optimal is the focus of the quickly emerging discipline of inverse optimization in operations research. Inverse optimization infers model coefficients from observed decisions, in contrast to traditional optimization, which seeks the optimal solution for the given known parameters. This Paper demonstrate the implementation of inverse optimization through a case study in supply chain cost estimation. The inverse problem is developed to extract transportation cost coefficients from observable shipping patterns using a linear programming transportation model. The formulation in mathematics, the method of solving it, and the computing example are presented. The paper emphasises how inverse optimization is useful in decision sciences, logistics, energy systems, and economics.

Keywords: Inverse optimization, inverse linear programming, supply chain, cost estimation, transportation problem.

INTRODUCTION

Optimization models are widely used in engineering, economics, finance, and management sciences to determine optimal decisions under given constraints. In a Classical optimization model, it is assumed that model parameters (costs, weights, utilities) are known and fixed. However, in many real-world problems, these model parameters are unknown or uncertain, but decisions are observable. These problems can be tackle by inverse optimization.

In an Inverse optimization model, we can estimate the model parameters that makes the observed decision optimal. Burton and Toint [1] were the first who investigate the inverse optimization for shortest path problem. Ahuja and Orlin[2] formally proposed the idea of inverse optimization in the field of network flow problems and linear programming problem, since then a lot of work has been done in various field such as ,minimum cost flow problem[9], minimum spanning tree problem[10], shortest path problem[11,12], linear programming problem[2], Linear Fractional Programming problems[6,8], Quadratic Programming Problems [7] and Transportation Problems [3,5] etc.

In this paper, we have consider a case study in supply chain transportation, where we have given the shipment quantity(observed solution) and we wish to estimate the transportation costs for which the observed solution become optimal.

Case Study: Transportation Cost Estimation

Problem Description

A company distributes goods from two warehouses W_1, W_2 to three retail outlets R_1, R_2 and R_3. The shipment quantities are observed, but transportation costs are unknown.

Observed shipment from warehouses W_1,W_2 to retail outlets R_1 ,R_2 and R_3 are given in the following matrix:

Table: 1

From/ To	R_1	R_2	R_3
W_1	$X_{11} = 20$	$X_{12} = 30$	$X_{13} = 10$
W_2	$X_{21} = 10$	$X_{22} = 20$	$X_{23} = 40$

Supply from warehouses W_1, W2 are: $s_1 = 60$, $s_2 = 70$
Demands of R_1, R2 and R_3 are: $d_1 = 30$, $d_2 = 50$, $d_3 = 50$
Clearly, Total Demand= Total Supply
Supply and demand constraints are satisfied.

Management suspects that decisions were cost-optimal, but cost coefficients are unknown. The objective is to estimate transportation costs.

Method
Let the initial estimation of cost matrix is given as:

Table: 2

From/ To	R_1	R_2	R_3
W_1	$C_{11} = 8$	$C_{12} = 6$	$C_{13} = 10$
W_2	$C_{21} = 9$	$C_{22} = 7$	$C_{23} = 4$

Mathematical model of this problem is given as:

$$Minimize\ Z=\sum_{i=0}^{2}\sum_{j=0}^{3} c_{ij}x_{ij}$$

Subject to,
Supply constraints

$$x_{11}+x_{12}+x_{13}=60$$
$$x_{21}+x_{22}+x_{23}=70$$

and demand constraints

$$x_{11}+x_{21}=30$$
$$x_{12}+x_{22}=50$$
$$x_{13}+x_{23}=50$$

$$x_{11}\geq 0 \text{ for } i=0,1,2\ ;\ j=0,1,2,3$$

If u_1, u_2 are dual variables correspond to supply constraints and v_1, v_2, v_3 are dual variables correspond to demand constraints then the dual of the above problem is :

$$Maximixe\ W=60u_1+70u_2+30v_1+50v_2+50v_3$$

Subject to,

$$u_1+v_1\leq c_{11}$$
$$u_1+v_2\leq c_{12}$$
$$u_1+v_3\leq c_{13}$$
$$u_2+v_1\leq c_{21}$$
$$u_2+v_2\leq c_{22}$$
$$u_2+v_3\leq c_{23}$$

Here the observed shipment are positive i.e. all $x_{ij}>0$ therefore by complementary slackness conditions:

$$u_1+v_1=c_{11}$$
$$u_1+v_2=c_{12}$$
$$u_1+v_3=c_{13}$$
$$u_2+v_1=c_{21}$$
$$u_2+v_2=c_{22}$$
$$u_2+v_3=c_{23}$$

Let $C^0=\begin{bmatrix}8 & 6 & 0\\ 9 & 7 & 4\end{bmatrix}=[c_{ij}^0]$ be the initial estimation of the cost and using inverse optimization, we have to find the cost $C=[c_{ij}]$ for which the observed shipment become the optimal shipment and both c_{ij} and c_{ij}^0 are differ as little as possible.

Therefore we have to minimize $\sum_{i,j}(c_{ij}-c_{ij}^0)^2$. Substituting $c_{ij}=u_i+v_j$, and the values of c_{ij}^0, we have the following function to minimize:

f(u,v) = $(u_1+v_1-8)^2+(u_1+v_2-6)^2+(u_1+v_3-10)^2+(u_2+v_1-9)^2+(u_2+v_2-7)^2+(u_2+v_3-4)^2$

Calculating partial order derivatives $\frac{\partial f}{\partial u_1},\frac{\partial f}{\partial u_2},\frac{\partial f}{\partial v_1},\frac{\partial f}{\partial v_2},\frac{\partial f}{\partial v_3}$ and set equal to zero we get the following equations:

$$3u_1+v_1+v_2+v_3=24 \quad \text{(i)}$$
$$3u_2+v_1+v_2+v_3=20 \quad \text{(ii)}$$
$$u_1+u_2+2v_1=17 \quad \text{(iii)}$$
$$u_1+u_2+2v_2=13 \quad \text{(iv)}$$
$$u_1+u_2+2v_3=14 \quad \text{(v)}$$

Subtracting (ii) from (i), we get
$u_1-u_2=4/3$

Subtracting (iv) from (iii), and (v) from (iii), we get
$v_1-v_2=2$ and $v_1-v_3=1.5$
or $v_2=v_1-2$ and $v_3=v_1-1.5$

Substituting v_2 and v_3 in (i), we have
$3u_1+3v_1=27.5$ or $u_1+v_1=9.167$

again Subtracting (v) from (iii), we get
$v_1-v_3=1.5$
In order to remove degeneracy, we have to fix one variable. If we start from $v_1=5$, the we get u_1 = 4.167, u_2 = 2.833, v_2 = 3 and v_3 = 3.5.

Further, using $c_{ij}=u_i+v_j$ the estimated cost can be calculated and the following is the estimated cost matrix:
$$C=\begin{bmatrix}9.167 & 7.167 & 7.667\\ 7.833 & 5.833 & 6.333\end{bmatrix}$$

LIMITATIONS

- It requires feasibility of observed solution
- Inverse problem usually a complex problem having large number of variables
- Inverse problem may have multiple possible solutions.

CONCLUSION

Inverse optimization is a powerful methodology bridging optimization theory and data analytics. By inferring model parameters from observed decisions, it enables robust decision modelling in uncertain environments. The transportation case study demonstrates practical feasibility and managerial relevance.

REFERENCES

[1]. Burton, D. and Toint, Ph.L. (1992). On an instance of the inverse shortest paths problem. Mathematical Programming, 53, 45-61.

[2]. Ahuja, R. K. and Orlin, J. B. (2001). Inverse Optimization. Operations Research, 49, 771-783.

[3]. Jain,S., Arya,N. (2013). An Inverse Capacitated Transportation Problem. IOSR Journal of Mathematics, 5(4), 24-27.

[4]. Jain,S., Arya,N. (2013). Inverse Optimization for Quadratic Programming. Problems. International Journal of Operations Research Nepal, 2, 49-56.

[5]. Jain,S., Arya,N. (2013). An Inverse Transportation Problem with the Linear Fractional Objective Function Problems. AMO- Advanced Modeling and Optimization, 15(3), 677-687.

[6]. Jain,S., Arya,N. (2013). A Type of Inverse Linear Fractional Programming Problem. Mexican Journal of Operations Research, 2(2), 2-9.

[7]. Jain,S., Arya,N. (2013).On Inverse Quadratic Programming Problems. Advances in Modelling Series A. General Mathematics. 50(2), 55-71

[8]. Jain,S., Arya,N. (2012). Inverse linear fractional Programming: A new approach. Journal of Computer and Mathematical Sciences, 3(5), 542-547.

[9]. Zhang, J., Liu, Z. (2002). A general model of some inverse combinatorial optimization problems and its solution method under l_∞ norm. *Journal of Combinatorial Optimization*, 6, 207–227.

[10]. Zhang, J., Liu, Z. and Ma, Z. (1996). On the inverse problem of minimum spanning tree with partition constraints. *ZOR Math. Methods Oper. Res.*, 44, 171-188.

[11]. Zhang, J., Ma, Z. (1996). A network flow method for solving some inverse combinatorial optimization problems. *Optimization: A Journal of Mathematical Programming and Operations Research*, 37(1), 59-72.

[12]. Zhang, J. and Ma, Z. (1999). Solution structure of some inverse combinatorial optimization problems. *J. Comb. Optim.*, 3, 127–139.